Section 4.2 SUMS

Definition The finite sum $\sum_{i=1}^{n} f(i)$ is defined as

$$\sum_{i=1}^{n} f(i) = f(1) + f(2) + f(3) + \ldots + f(n)$$

Remarks 1) Here $\sum$ is called the sum (or sigma) notation.
2) "i" is called the index of summation. It is a "dummy variable" meaning you could use any letter instead of "i" of your choice.
3) The sum does not necessarily start at 1, it could start at zero or some other integer m and you would define the sum in the similar fashion
$\sum_{i=m}^{n} f(i) = f(m) + f(m+1) + f(m+2) + \ldots + f(n)$
4) The sum also could be written alternatively $\sum_{i=1}^{n} f(i) = \sum_{i=1}^{n} a_i$ where $f(i) = a_i$
Example

$$\sum_{j=1}^{5}(2j^2-1) = (2\cdot 1^2-1)+(2\cdot 2^2-1)+(2\cdot 3^2-1)+(2\cdot 4^2-1)+(2\cdot 5^2-1) = 105$$

Example

$$\sum_{i=1}^{15} 3 = \underbrace{3+3+\ldots+3}_{15-many} = 3\times 15 = 45$$

Example Let c be any arbitrary constant

$$\sum_{k=1}^{n} c = \underbrace{c+c+c+\ldots+c}_{n-many} = n\times c = nc$$

Theorem For constant c and d,

$$\sum_{i=1}^{n}(c\cdot a_i + d\cdot b_i) = c\sum_{i=1}^{n} a_i + d\sum_{i=1}^{n} b_i$$