Section 4.1 Anti Differentiation

Definition: If $\frac{dF(x)}{dx} = f(x)$ then we say that F(x) is **an anti-derivative** of f.

Example 1) $F(x) = \frac{x^3}{3}$ is an anti-derivative of $f(x) = x^2$ because $\frac{dF}{dx} = \frac{3x^2}{3} = x^2 = f(x)$

2)$F(x) = x \ln x - x$ is an anti-derivative of $f(x) = \ln x$. Show this yourself by taking derivative of F(x)

3)$F(x) = \frac{x^2}{2}$ is an anti-derivative of $f(x) = x$.Show this yourself by taking derivative of F(x)

4)$F(x) = \frac{x^2}{2} + 1$ is an anti-derivative of $f(x) = x$

4)$F(x) = \frac{x^2}{2} + c$ is an anti-derivative of $f(x) = x$

Theorem If $F'(x) = f(x)$ and $G'(x) = f(x)$ then $F(x) = G(x) + c$ for some constant factor c.

Proof $F'(x) = f(x)$ and $G'(x) = f(x)$ implies that $(G-F)' = f - f = 0$. Recall that we proved as a corollary of the Mean Value Theorem that if a function has a derivative zero then it is constant. Hence $G(x) - F(x) = c$ (for some constant c). That is,$G(x) = F(x) + c$

Definition All anti-derivatives of a continuous function form a family $F(x) + c$ where c is an arbitrary constant. Thus the family $F(x) + c$ is called the <u>Indefinite Integral</u> of f and we write

$$\int f(x)\, dx = F(x) + c$$

<u>Remark</u> ”x” in the sign $\int f(x)\, dx$ is not a dummy variable.