

Theorem to find the "w" : $12^2 + y^2 = 13^2 \Rightarrow y = 5$. Now we have everything to evaluate the derivative:

$$\frac{dy}{dt} = \frac{-12}{5} \cdot (5) = -12 \text{ ft/sec}$$

Again the negative sign shows that the ladder is moving down.

b) Consider the triangle formed by the ladder, wall and the floor. Find the rate at which the area of the triangle is changing when the base of the ladder is 12 ft from the wall?

The area of the triangle is $A = \frac{1}{2}x \cdot y$ based on our figure in part(a). So for this problem we have a readily available "connection" equation and we will differentiate both sides of this equation with respect to time. Note that we need to use the Product Rule on the right hand side and since both w and f are changing with respect to time we should not forget $\frac{dy}{dt}$ or $\frac{dx}{dt}$ that comes because of the differentiation of a w or an f term.

$$\frac{dA}{dt} = \frac{1}{2}x \frac{dy}{dt} + \frac{1}{2}y \frac{dx}{dt}$$

We know that $x = 12$ and $y = 5$ from the previous part. Also $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = -12$ from above. Putting this altogether we get $\frac{dA}{dt} = \frac{1}{2}12(-12) + \frac{1}{2}5(5) = -\frac{119}{2} \text{ ft}^2/\text{sec}$. Once again the negative here means that the area is decreasing, which makes sense since the ladder is falling and creating a smaller triangle.