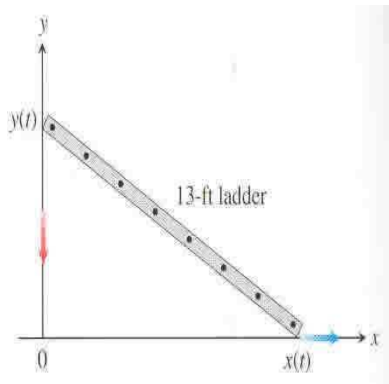
Example a) A 13ft long ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 5 ft per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 12ft from the wall.



As seen in the figure the length of the ladder is 13ft; and "y" represents the height of the ladder on the wall at certain time t and "x" represents distance between the feet of the ladder and the wall. Both "y" and "x" are changing with time as the ladder slides down. We are also given $\frac{dx}{dt} = 5ft/sec$. We are asked to find the "speed" of the top of the ladder in this question as well. As in the previous example speed is the rate of change in the height (or distance to the ground) of the top of the ladder with respect to time. So we want to find $\frac{dy}{dt}$ when $x = 12ft$. To calculate this derivative we again need an equation that relates "y" to the other variable "x" and the ladder's length. At this point geometry becomes handy again and by using Pythagorean Theorem we write

$$x^2 + y^2 = 13^2$$

Now we will implicitly differentiate both sides of this equation with respect to time. Note that both variables w and f on both sides change with respect to time. This means when you do the differentiation expect $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to pop-up every time you are differentiating an x or y term.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

Note that we have a "y" on the right hand side of this equality. Since we are asked to find $\frac{dy}{dt}$ when $x = 12ft$, we will use the triangle and Pythagorean