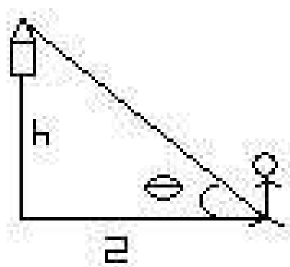
Example A rocket is launched vertically upward from a point 2 miles west of an observer on the ground. What is the speed of the rocket when the angle of elevation of the observer's line of sight to the rocket is $50°$ and is increasing at $5°$ per second?

If possible, always start with a graph or picture like the one below:



In the figure above the base b represents the distance of the observer to the rocket's launch pad and it is equal to 2. "h" represents the distance of the rocket to the ground and θ represent the angle of elevation of the observer's line of sight. Both h and θ are increasing in time. We are also given $\frac{d\theta}{dt} = 5°$ per second. We are asked to find the speed of the rocket when $\theta = 50°$. Speed is defined as the rate of change in distance of the rocket to the ground with respect to time i.e we are looking for $\frac{dh}{dt}$ when $\theta = 50°$. To find this we need an equation that relates h to the other variable θ and the base $b = 2$. This equation we will gather using the geometry. In the given figure above we have a right angular triangle hence we will use the trigonometry to connect h to others as follows

$$\tan\theta = \frac{h}{2}$$

Now we will implicitly differentiate both sides of this equation with respect to time. Note that both variables h and θ on both sides change with respect to time. This means when you do the differentiation expect $\frac{dh}{dt}$ and $\frac{d\theta}{dt}$ to pop-up every time you are differentiating an h or θ term.

$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{2}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 2\sec^2\theta\frac{d\theta}{dt}$$

Warning: Calculus requires that all angles be in radians.

$$\frac{dh}{dt} = 2\sec^2(50° \cdot \frac{\pi}{180})[5° \cdot \frac{\pi}{180}] \approx 0.422 mi/sec \approx 1520.7 mi/hr$$