

Let S denote the surface area of this cylindrical can then

$$S = 2\pi \cdot r^2 + 2\pi \cdot r \cdot h$$

because the top and bottom of can have surface area of $\pi \cdot r^2$ each because they are circles and the second term is the surface area of the side of the can which is a rectangle with *width* = h and *length* = $2\pi \cdot r$. The constraint equation for this problem is the volume info we are given, volume of the can –which we will denote by V – is 1000cm^3 (=1 liter). Also I know by the front of the book the volume of a cylinder is $V = h \cdot \pi \cdot r^2$ implies the constraint equation $h \cdot \pi \cdot r^2 = 1000$. Now solve this equation for h : $h = \frac{1000}{\pi r^2}$ and plug it into the surface area equation to turn S into a function of one variable:

$$S(r) = 2\pi \cdot r^2 + 2\pi \cdot r \cdot \left(\frac{1000}{\pi r^2}\right) = 2\pi r^2 + \frac{2000}{r}$$

The only sure restriction for the variable radius(r) I can come up with is; $r > 0$. So we are considering to maximize the surface area over the domain $(0, \infty)$. The handy rule above also suggested that with these kind of problems (fixed volume/optimize surface area) this kind of domain is no surprise. So I'll apply the One-critical value theorem again since my S is continuous on this interval. So let's find the critical value(s):

$$S'(x) = 4\pi r = \frac{2000}{r^2} = 0 \Rightarrow 4\pi r = \frac{2000}{r^2} \Rightarrow r^3 = \frac{500}{\pi} \Rightarrow r = \sqrt[3]{\frac{500}{\pi}}$$

Finding only one critical value is promising. But I still have to show the sign change at this r to conclude that it gives me the absolute minimum for the surface area. I'll leave this to you as an exercise to make the sign chart and show that at $r = \sqrt[3]{\frac{500}{\pi}}$ S has a minimum. Hence when the radius is $r = \sqrt[3]{\frac{500}{\pi}} \approx 5.42\text{cm}$ S has an absolute minimum. Then the height is $h = \frac{1000}{\pi(\sqrt[3]{\frac{500}{\pi}})^2} \approx 58.74\text{cm}$