One-Critical-Value Extrema Theorem Suppose f(x) is a continuous function defined on (a, b) (where here $a = -\infty$ and $b = \infty$ are possible). If $f'(c) = 0$ or undefined and $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$ then f has an absolute minimum (in its domain) at c. If $f'(c) = 0$ or undefined and $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$ then f has an absolute maximum (in its domain) at c.

Remarks 1) Notice that this is very similar to the 1st derivative test for local extrema. The difference is that in this test we require information about the sign of the derivative for all $x < c$ and $x > c$; in the local test, we needed to only know this information for values of x near c.

2) To use this theorem for absolute extrema directly, you should be in the situation where you **have exactly one critical point in your domain**. If you have more than one critical point in your domain, you probably have to do more work.

Example Find two numbers whose sum is 23 and whose product is maximum.

Normally we would start a problem like this drawing a picture. For this problem, though, there is no real need. We see were looking for numbers x and y such that $x + y = 23$ and so that their product P is maximized: $P = x \cdot y$. Notice in particular that x and y have no further restriction. In particular, x could be any real number. Now since $x + y = 23$, we have $y = 23 - x$. Hence were trying to maximize

$$P(x) = x(23 - x) = 23x - x^2$$

on the domain $(-\infty, \infty)$. So we will use One-critical-value-extrema theorem.
$P'(x) = 23 - 2x = 0 \Rightarrow x = \frac{23}{2}$
To find the sign of the derivative for values $x < 23/2$ it is enough to compute the sign of the derivative for one value of $x < 23/2$. Similarly, to test the sign of the derivative for values $x > 23/2$ it is enough to compute the sign of the derivative for one value $x > 23/2$. So we may use our sign chart below