polynomial, it certainly is continuous. so we may use the EVT. So let's find critical points for A(u). Since

$$A'(u) = \frac{1}{8}u - \frac{\sqrt{3}}{18}(12 - u)$$

we see first that the derivative is defined everywhere so that critical points are simply zeroes of the first derivative. When is $A'(u) = 0$?
$\frac{1}{8}u + \frac{\sqrt{3}}{18}u = \frac{12\sqrt{3}}{18} \Rightarrow u \approx 5.21$

Finally, we evaluate the function at these critical points to find the maximum and minimum

u	A(u)
0	$A(0) \approx 6.93$
5.21	$A(5.21) \approx 3.92$
12	$A(12) \approx 9$

From the table we see that area is maximized when $u = 12$ (that is, when all of the wire is used to make the square) and minimized when 5.21 meters is used to make the square (and so 6.79 meters is used to make a triangle).

Optimization on open intervals

It often happens that you are asked to optimize a function on an open interval (or the whole real line) instead of a closed interval? How does one go about solving such a problem? Happily, much of the technique in optimizing functions on closed intervals will carry over to optimizing functions on open intervals. The main difference is that in finding absolute maxima or minima we will use a variation of the 1st derivative test instead of our techniques for finding absolute maxima and minima for continuous functions on closed intervals. This variation of the 1st derivative test is: