Optimization on closed intervals

Example A farmer has a 2,400 ft of fencing and wants to fence off a rectangular field that borders a straight river. (No fence needed along the river side). What are the dimensions of the field that has the largest area?

Here is my solutions process: After I have read the problems a few times to understand what it's asking I know that I'm asked to maximize the area(A) of a rectangular region. I also know that the total fence length is 2,400 ft. I do not know the length and the width of the rectangular field.

Then I like to draw a picture of whats going on. In the picture Ill label certain quantities so I can keep track of whats happening. Below W represents the width and L represents the length of the field



I will maximize the area and the area of a rectangle is $A = L \cdot W$. Perhaps a less obvious thing to write down is what I call a constraining equation; its an equation that remembers how the variables W and L relate to each other. In this case, the relationship is: 2W + L = 2400

Because the total fencing available will be used to cover the three sides of this farmland. The total length of these sides is W+W+L.

What I'd like to do next is to take my expression for area and optimize it. The problem is that; currently it sits as a function of two variables instead of one. We don't have the technology yet to optimize a function of 2 variables. So what do we do? We will take our constraining equation and use it to solve for L in terms of W: $2W + L = 2,400 \Rightarrow L = 2,400 - 2W$.