Example Sketch the graph of $f(x) = \frac{3}{4}(x^2-1)^{2/3}$

Domain f(x) is is defined for all real numbers. So Domain of f=$\Re$

Intercepts y-intercept: $\frac{3}{4}(0^2-1)^{2/3} = \frac{3}{4} \Rightarrow (0, \frac{3}{4})$
x-intercept: $\frac{3}{4}(x^2-1)^{2/3} = 0 \Rightarrow x^2 = 1 \Rightarrow (\pm 1, 0)$.

Asymptotes

Vertical Asymptote Since f has no problem points in its domain we do not have any vertical asymptotes.

Horizontal Asymptote $\lim\limits_{x\to\infty} \frac{3}{4}(x^2-1)^{2/3} = \infty$. Also $\lim\limits_{x\to-\infty} \frac{3}{4}(x^2-1)^{2/3} = \infty$ says there is no horizontal asymptotes either.

Critical Values Find the derivative and determine where it is zero or undefined.

$$f'(x) = (\frac{\not{3}}{\not{4}})(\frac{\not{2}}{\not{3}})(x^2-1)^{-1/3}(\not{2}x) = \frac{x}{(x^2-1)^{1/3}}$$

Now the critical values
$f'(x) = 0 \Rightarrow x = 0$ and $f'(x)$ dne when $x^2 = 1 \Rightarrow x = \pm 1$.
So critical values are $x = -1, 0, 1$

2nd Derivative Resolve the concavity issue finding the second derivative

$$f''(x) = \frac{(x^2-1)^{1/3} - x\frac{1}{3}(x^2-1)^{-2/3}(2x)}{(x^2-1)^{2/3}} = \frac{x^2-3}{3(x^2-1)^{4/3}}$$

$f''(x) = 0$ implies $x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ and $f''(x)$ dne implies $x = \pm 1$