Example Sketch the graph of $f(x) = \frac{x^2}{\sqrt{x+1}}$

$\boxed{Domain}$ f(x) is not defined when $\sqrt{x+1} \leq 0$. So we want $x+1 > 0 \Rightarrow x > -1$ which implies Domain of f=$(-1, \infty)$

$\boxed{Intercepts}$ y-intercept: $\frac{0^2}{\sqrt{0+1}} = 0 \Rightarrow (0,0)$ /x-intercept: $\frac{x^2}{\sqrt{x+1}} = 0 \Rightarrow x = 0 \Rightarrow (0,0)$. So there is only one intercept.

$\boxed{Asymptotes}$

Vertical Asymptote f is not defined at $x = -1$ but we know we need to check the limit to make sure that it is indeed the vertical asymptote. Since $\lim\limits_{x\to 1^+} \frac{x^2}{\sqrt{x+1}} = \infty$, $x = -1$ is a vertical asymptote.
Note that I only have to check one sided limit here as f is not defined on the other side.

Horizontal Asymptote For the horizontal asymptote we need the limit at ∞

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x+1}} \underbrace{=}_{L'Hop} \lim_{x\to\infty} \frac{2x}{\frac{1}{2\sqrt{x+1}}} = \lim_{x\to\infty} x\sqrt{x+1} = \infty$$

So f has no horizontal asymptote.

$\boxed{Critical \quad Values}$ Find the derivative and determine where it is zero or undefined.

$$f'(x) = \frac{2x(\sqrt{x+1}) - x^2(\frac{1}{2\sqrt{x+1}})}{(\sqrt{x+1})^2} = \frac{3x^2+4x}{2(x+1)^{3/2}}$$

Now the critical values $f'(x) = 0 \Rightarrow 3x^2 + 4x = 0 \Rightarrow x(3x+4) = 0 \Rightarrow$
$x = 0$ or $x = \frac{-4}{3}$. Since $x = -\frac{4}{3}$ is not in our domain we will exclude this one from out sign chart.
$f'(x)$ dne when $x = -1$ but then again this point is not in our domain.

$\boxed{2nd \quad Derivative}$ Resolve the concavity issue finding the second derivative

$$f''(x) = \frac{(6x+4)[2(x+1)^{3/2}] - (3x^2+4x)[3(x+1)^{1/2}]}{[2(x+1)^{3/2}]^2} = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$$