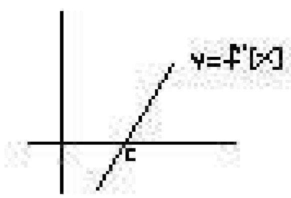
Second Derivative Test For a function f let f'' exists and be continuous on an interval (a, b) and f be continuous on $[a, b]$. Then if c in (a, b) is a critical point of f, then
a) If $f''(c) > 0$ then f has a local minimum at $x = c$
b) If $f''(c) < 0$ then f has a local maximum at $x = c$
a) If $f''(c) = 0$ then you cannot make a conclusion about c.

Remarks 1)The upside to this test is that it is very quick (provided you have the second derivative) to show whether a critical point is a max or min. The downside is that it might happen that $f''(c) = 0$. In this situation, one cannot conclude from the second derivative alone whether f has a max or a min at c.

Why does the test work? Well, if $f''(c) > 0$ this means $f'(x) \nearrow$ on a neighborhood around c. Also since c is a critical point $f'(c) = 0$. So the graph of $f'(x)$ around c looks like roughly like the graph below



So f' changes sign from negative to positive which is by the first derivative test is where we have a local minimum.

Similarly, if $f''(c) < 0$ this means $f'(x) \searrow$ on a neighborhood around c. Also since c is a critical point $f'(c) = 0$. So the graph of $f'(x)$ around c looks like roughly like the graph below