Example Determine where the function $F(x) = |x^2 - 1|$ is concave up and/or concave down? Also find points of inflection. Then using two derivative tests we have given so far give a rough sketch of $F(x)$

To find the derivatives of F we need to re-write F as follows

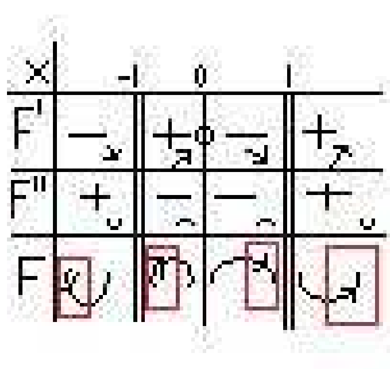
$$F(x) = |x^2-1| = \begin{cases} x^2 - 1 & \text{if } x^2 \geq 1 \\ -x^2+1 & \text{if } x^2 < 1 \end{cases} = \begin{cases} x^2 - 1 & \text{if } x \leq -1 \text{ or } x \geq 1 \\ -x^2+1 & \text{if } -1 < x < 1 \end{cases}$$

So $F'(x) = \begin{cases} 2x & \text{if } x < -1 \text{ or } x > 1 \\ dne & \text{if } x = -1 \text{ or } x = 1 \\ -2x & \text{if } -1 < x < 1 \end{cases}$

Since we will need the information from first derivative test to graph F let's find its critical values first. Clearly, $x = \pm 1$ are critical points as F' does not exist there. Also $F'(x) = 0$ implies $-2x = 0$ implies $x = 0$ is another critical point. I will postpone the sign chart construction to later for reasons which will become soon clear. Next the second derivative

$$F''(x) = \begin{cases} 2 & \text{if } x < -1 \text{ or } x > 1 \\ dne & \text{if } x = -1 \text{ or } x = 1 \\ -2 & \text{if } -1 < x < 1 \end{cases}$$

So $F''(x)$ is never equal to zero but it fails to exist at $x = \pm 1$. I will next create a sign chart that gathers information from first and second derivative together. I'll explain after the table how the choice about the shape of F is made on each interval



The red rectangle in each of the boxes indicates the shape of F on that interval. How is it determined: On the interval $(-\infty, -1)$ the first derivative is negative, so F is $\searrow$; the second derivative on this interval is positive so