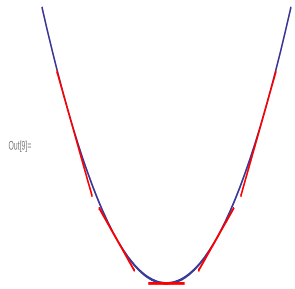

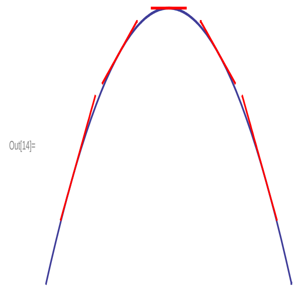

Note that the tangent lines drawn to first graph are all below the graph. Also the slopes of tangent lines are increasing as x moves from left to right. Whereas in the second graph all the tangent lines are above the graph. Also the slopes of tangent lines are decreasing as x moves from left to right. So here is the definition;

Definition If the graph of f lies

(i) above all of its tangents on an interval I, then it is *concave upward* on I.
(ii) below all of its tangents on an interval I, then it is *concave downward* on I.

Remarks • By the discussion just before the definition we know that if f is concave upward then the slope's of the tangent lines is increasing, this means f' is increasing. Hence the derivative of f' –which is f'' – is positive on the interval f is concave up.

• Similarly, if f is concave downward then the slope's of the tangent lines is decreasing, this means f' is decreasing. Hence the derivative of f' –which is f'' – is negative on the interval f is concave down.

Theorem(The Concavity Test) a) If $f''(x) > 0$ for all x in an interval I, then the graph of f is **concave up** on I.

b) If $f''(x) < 0$ for all x in an interval I, then the graph of f is **concave down** on I.