

First Derivative Test Let $f(x)$ be continuous on the interval $[a, b]$ and c a critical point of $f(x)$ on the open interval (a, b) , then

1) $f(c)$ is a local maximum if

$$f'(x) > 0 \text{ for all } x \text{ in } (a, c) \text{ and } f'(x) < 0 \text{ for all } x \text{ in } (c, b).$$

2) $f(c)$ is a local minimum if

$$f'(x) < 0 \text{ for all } x \text{ in } (a, c) \text{ and } f'(x) > 0 \text{ for all } x \text{ in } (c, b).$$

3) $f(c)$ is not a local extremum if $f'(x)$ has the same sign on (a, c) and (c, b) .

Example Find where the function $F(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and decreasing and classify the critical points of F .

We have to first find the critical points. Since $F(x)$ is a polynomial the derivative does not exist case is not a possibility. So we will only explore $F'(x) = 0$ one.

$$F'(x) = 0 \Rightarrow 12x^3 - 12x^2 - 24x = 0 \Rightarrow 12x(x^2 - x - 2) = 0 \Rightarrow 12x(x+1)(x-2) = 0$$

So the critical values are $x = 0$, and $x = -1$ and $x = 2$. To classify these critical points we'll use the First Derivative Test. It is much easier to observe the test in action on a sign chart.

x	-1	0	2
F'	-	+	-
F	↘	↗	↘