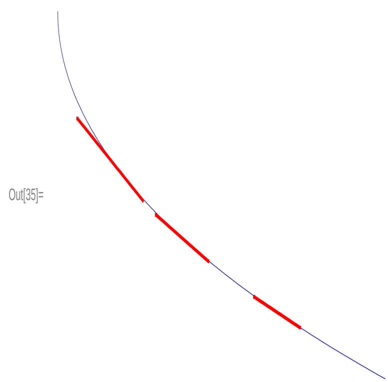
Next we have a graph of a function that is decreasing. It also has the tangent lines drawn to it at several points. Note that the slopes of these tangent lines are all negative. Hence f' appears to be negative everywhere.



So the following theorem should come as no surprise:

Theorem Suppose that f is a differentiable function on an interval I

1) If $f'(x) > 0$ for all x in I, then f is increasing($\nearrow$) on I.
2) If $f'(x) < 0$ for all x in I, then f is decreasing($\searrow$) on I.

Proof We have proved the first result as a corollary of Mean Value Theorem in class. Here to remind ourselves MVT we will prove the second one:
Let $f'(x) < 0$ on the interval I. Pick any two points x_1 and x_2 in I where $x_1 < x_2$. Applying MVT on the interval $[x_1, x_2]$ (note that this small interval in contained in our bigger interval I) because f is continuous and differentiable on I hence on $[x_1, x_2]$, we know that there is a c in the interval $[x_1, x_2]$ such that

$$(\star) \quad f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

By our assumption $f'(c) < 0$ also $x_2 - x_1 > 0$ because $x_1 < x_2$ implies by the $(\star)$ above that $f(x_2) - f(x_1) < 0$. Hence $f(x_2) < f(x_1)$. Since our choices of x_1 and x_2 are arbitrary this result will hold for all possible choices in I. So for any $x_1 < x_2$, $f(x_1) > f(x_2)$. By definition then f is decreasing on I.

This theorem suggests that we can gather information about $f(x)$ using $f'(x)$. The next theorem will give the full recipe of how to do exactly that. To make sure you have a better understanding of the next result we will