this, notice that we could make a rectangle with no height (so that $x = 0$). This would be dumb, but we could do it. We could make another dumb rectangle: one without width, so that $x = 5$ (think about why this is 5 and not 10). The possible rectangles we could construct sit somewhere between these two stupid extremes, so we see that x lives in the interval $[0, 5]$. Now we are really in business since we have reduced our original problem to the following: Maximize the function $A(x) = 5x - x^2$ on the interval $[0, 5]$.

Since $A(x)$ is continuous on $[0, 5]$ (it's a polynomial, so in fact its continuous everywhere), we can find the extreme values of A by computing the value of $A(x)$ at critical points and endpoints. We proceed to find critical points: $A'(x) = 5 - 2x$, and so the only critical point we have is $x = \frac{5}{2}$. Now we need to evaluate A(x) at the critical point and the endpoints of the interval.

x	f(x)
0	$A(0) = 0$
$\frac{5}{2}$	$A(\frac{5}{2}) = \frac{25}{4}$
5	$A(5) = 0$

So to maximize the area of our rectangle we will make $x = \frac{5}{2}$ (and so $y = \frac{5}{2}$ since $y = 5 - x$), and the resultant area will be $\frac{25}{4}$. We can restate our result: of all rectangles with a fixed perimeter, a square maximizes area.