

$$\begin{aligned}
\frac{d}{dx}x\sqrt{x-x^2} &= \sqrt{x-x^2} + x\frac{1}{2\sqrt{x-x^2}}(1-2x) \\
&= \sqrt{x-x^2} + \frac{x-2x^2}{2\sqrt{x-x^2}} \\
&= \frac{2(x-x^2) + (x-2x^2)}{2\sqrt{x-x^2}} \\
&= \frac{3x-4x^2}{2\sqrt{x-x^2}}
\end{aligned}$$

Now critical points of  $f$  are where  $f'(x) = 0$  or when  $f'(x)$  is undefined. Zeros of  $f'(x)$  occur when the numerator of the derivative is zero.

$$3x - 4x^2 = x(3 - 4x) = 0 \Rightarrow x = 0 \text{ or } x = \frac{3}{4}$$

And  $f'(x)$  is undefined when the denominator is 0, this happens

$$x - x^2 = x(1 - x) = 0 \Rightarrow x = 0 \text{ or } x = 1$$

So the critical values are  $x = 0, 3/4, 1$ . Next I will combine the last two steps: We will evaluate  $f(x)$  at the critical points and endpoints of the interval and then compare:

x	f(x)
0	$f(0) = 0\sqrt{0-0^2} = 0$
$\frac{3}{4}$	$f(3/4) = \frac{3}{4}\sqrt{\frac{3}{4} - (\frac{3}{4})^2} > 0$
1	$f(1) = 1\sqrt{1-1^2} = 0$

Since 0 is the smallest value we see for  $f(x)$  in this chart, we have that 0 and 1 are both absolute minima of  $f(x)$ . Since  $f(3/4) > 0$  this means  $x = 3/4$  is an absolute maximum of the function  $f(x)$ .

**Example** Find the absolute maxima and minima of the function  $f(x) = |x|$  on the interval  $[-1, 2]$ .

First absolute value function is continuous on all of  $\mathfrak{R}$ , so it is continuous on  $[-1, 2]$ . Next we need the derivative of  $f(x)$ . Recall that

$$f'(x) = \begin{cases} -1 & \text{if } x < 0 \\ dne & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

So  $x = 0$  is the only critical value of  $f(x)$ . Next we need to evaluate  $f(x)$  at the critical point and endpoints of the interval.