

Theorem Suppose that f is continuous on the closed interval $[a, b]$. Then the absolute extrema of f occurs at an endpoint (a or b) or at a critical number.

For the proof of the last two theorems please check out your books. The proofs given are quite easily understandable. I'll be happy to answer any questions you have about them outside of the class.

The last theorem gives us the following recipe to find absolute minimum and maximum of a function on a closed interval.

Finding Absolute Maxima and Minima

- (i) Verify that $f(x)$ is continuous on a closed interval of interest.
- (ii) Find critical points of $f(x)$ that lie in the closed interval of interest
- (iii) Evaluate $f(x)$ at critical points and endpoints in a chart
- (iv) Pick out the absolute maximum by finding which critical point/endpoint gives the largest function value (similarly for absolute minimum)

Remark It is IMPORTANT if you're going to use this process that you verify the first step. If you don't first check that the function is continuous on a closed interval, this method can fail! Now let's apply it :

Example Find the absolute maxima and minima of the function $f(x) = x\sqrt{x-x^2}$ on the interval $[0, 1]$

We want to use the procedure above, so first we need to verify that $f(x)$ is continuous on $[0, 1]$. But since x is continuous everywhere and $\sqrt{x-x^2}$ is defined on the interval $[0, 1]$ (and hence continuous there) we see that $f(x)$ is continuous on $[0, 1]$ as desired.

For the second step, we need to find critical points of $f(x)$. This means we need to compute $f'(x)$. Now