Some other interesting results

We will use L'Hop to race the functions such as x, e^x, $\ln x$ and answer which one goes to infinity faster.

Example For $a > 0$,

$\lim_{x\to\infty} \frac{e^{ax}}{x} = \lim_{x\to\infty} \frac{ae^{ax}}{1} = \infty$ by L'Hop

So e^{ax} grows faster than x (for $a > 0$).

Example For $a > 0$, using L'Hop ten times below we evaluate the following limit

$$\lim_{x\to\infty} \frac{e^{ax}}{x^{10}} = \lim_{x\to\infty} \frac{ae^{ax}}{10x^9} = \lim_{x\to\infty} \frac{a^2e^{ax}}{10\cdot 9x^8} = \ldots = \lim_{x\to\infty} \frac{a^{10}e^{ax}}{10!} = \infty$$

Example $\lim_{x\to\infty} \frac{\ln x}{x^{1/3}} = \lim_{x\to\infty} \frac{1/x}{1/3x^{-2/3}} = \lim_{x\to\infty} 3x^{-1/3} = 0$

Combining the three preceding examples, $\ln x \ll x^{1/3} \ll x \ll x^{10} \ll e^{ax}$ (as $x \to \infty$, $a > 0$) (Here $a \ll b$ reads "a is much slower than b")