Other Indeterminate Forms

While calculation of limits you might also encounter other indeterminate forms such as $\infty - \infty$, $0 \cdot \infty$, 1^∞, 0^0, ∞^0. These can be manipulated into $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Example Find $\lim\limits_{x \to 0^+} x \ln x$

Now if you try to evaluate this limit you will get $0 \cdot (-\infty)$ indeterminate form. Re-writing the function as follows

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}$$

turns the indeterminate from into $\frac{-\infty}{\infty}$ so we may use L'Hop to continue.

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} \overbrace{=}^{L'Hop} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} -x = 0$$

On a side note you could have used $\frac{x}{\frac{1}{\ln x}}$ to change your indeterminate form but algebra with this choice is messy.

When dealing with indeterminate forms such as 1^∞, 0^0 or ∞^0 use the following remark

If a is a positive number then $a = e^{\ln a}$. *So if we have* $[f(x)]^{g(x)} > 0$, *then* $f(x)^{g(x)} = e^{\ln f(x)^{g(x)}} = e^{g(x) \ln f(x)}$ *and*
$\lim_{x \to a} f(x)^{g(x)} = \lim_{x \to a} e^{g(x) \ln f(x)} = e^{(\lim_{x \to a} g(x) \ln f(x))}$

Example Find $\lim\limits_{x \to 0^+} x^x$. Note that this limit has 0^0 indeterminate form. We will use the above remark to evaluate the limit

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{lnx^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{(\lim\limits_{x \to 0^+} x \ln x)} = e^0 = 1$$

Second to last equality is due to the previous Example.