Example Find $\lim\limits_{x\to 0}\frac{\cos x-1}{x^2}$

$$\lim_{x\to 0}\frac{\cos x-1}{x^2}\underbrace{=}_{(\frac{0}{0})L'Hop}=\lim_{x\to 0}\frac{-\sin x}{2x}\underbrace{=}_{(\frac{0}{0})L'Hop}=\lim_{x\to 0}\frac{-\cos x}{2}=-\frac{1}{2}$$

As in this example sometimes you might have to use "L'Hop" *more than once.*

Example Find $\lim\limits_{x\to\frac{\pi}{2}}\frac{\sec x}{\tan x}$

$$\lim_{x\to\pi/2}\frac{\sec x}{\tan x}\overbrace{=}^{(\frac{\infty}{\infty})L'Hop}\lim_{x\to\pi/2}\frac{\sec x\tan x}{\sec^2 x}=\lim_{x\to\pi/2}\frac{\tan x}{\sec x}\overbrace{=}^{(\frac{\infty}{\infty})L'Hop}=\lim_{x\to\pi/2}\frac{\sec^2 x}{\sec x\tan x}\overbrace{=}^{(\frac{\infty}{\infty})L'Hop}\lim_{x\to\pi/2}\frac{\sec x}{\tan x}$$

Note here that even though "L'Hop" is *applicable it is not useful.* It brought us back where we started. But using the trig we have a way out

$$\lim_{x\to\pi/2}\frac{(1/\cos x)}{(\sin x/\cos x)}=\lim_{x\to\pi/2}\frac{1}{\sin x}=1$$

Example Find $\lim\limits_{x\to 0^+}\frac{\sin x}{x^2}$

$$\lim_{x\to 0^+}\frac{\sin x}{x^2}\overbrace{=}^{L'Hop}=\lim_{x\to 0^+}\frac{\cos x}{2x}\overbrace{=}^{L'Hop}\frac{-\sin x}{2}=0$$

But this doesn't agree what we get from linear approximation:
$\frac{\sin x}{x^2}\approx\frac{x}{x^2}=\frac{1}{x}\to\infty$ as $x\to 0^+$

We can clear up this seeming conflict between the two approach by noting that $\lim\limits_{x\to 0^+}\frac{\cos x}{2x}=\frac{1}{0}$, so this limit is not of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ so we may not use "L'Hop". So *look before you L'Hop!*