Section 3.2 L'H$\hat{o}$pital's Rule

We have run into several indeterminate forms when we are dealing with the limits such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$. We have dealt so far with such forms using algebraic manipulations. Here is a recall example;

Example $\lim\limits_{x\to 1} \frac{x^3-1}{x^2-1} = \lim\limits_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim\limits_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{3}{2}$
In this section we will get introduced to a tool called $L'\hat{o}pital'sRule$ that will help us to evaluate such indeterminate limits by use of derivatives.

L'H$\hat{o}$pital's Rule If $f(x)$ and $g(x)$ are differentiable on (a, b) except possibly at some c in the interval (a, b) and $g'(x) \neq 0$ on (a, b) except possibly at c, and if $\lim\limits_{x\to c} \frac{f(x)}{g(x)}$ has indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\boxed{\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}}$$

provided the limit on the right hand side exists (or diverges to ∞ or $-\infty$).

Remarks
1) Before using "L'Hop" you have to make sure that you have the required indeterminate form first, meaning either $\frac{0}{0}$ or $\frac{\infty}{\infty}$. If the indeterminate form you have is a different one (such as 1^∞ or $\infty - \infty$) than these two you either have to find a way to convert it into one of these or use another method to evaluate the limit.

2) "L'Hop" applies to one sided limits as well as limits at infinity.

3) "L'Hop" is useful because it automatically factors and cancels the "bad" factors in the ratio $f(x)/g(x)$