Example Lets say we are on Planet Q, and that a satellite is whizzing overhead with a velocity v. We want to find the time dilation that the clock onboard the satellite experiences relative to my wristwatch. We borrow the following equation from special relativity

$$T_m = \frac{T}{\sqrt{1-\frac{v^2}{c^2}}}$$

Here, T_m is the time I measure on my wristwatch, and T is the time measured onboard the satellite.

$$T_m = \frac{T}{\sqrt{1-\frac{v^2}{c^2}}} \approx T + \frac{1}{2}\left(\frac{v^2}{c^2}\right)$$

Above approximation is using Number (5) from the list of approximations we have generated before with
$(1+u)^r \approx 1 + ru$ where $u = -\frac{v^2}{c^2}$, and $r = -\frac{1}{2}$

If $v = 4$ km/s, and the speed of light (c) is 3×10^5 km/s, $\frac{v^2}{c^2} \approx 10^{-10}$. There is hardly any difference between the times measured on the ground and in the satellite. Nevertheless, engineers used this very approximation (along with several other such approximations) to calibrate the radio transmitters on GPS satellites. (The satellites transmit at a slightly offset frequency.)

Example During the medical procedure, the size of a roughly spherical tumor is estimated by measuring its diameter and using the formula $V = \frac{4}{3}\pi R^3$ to compute its volume. If the diameter is measured as 2.5cm with a maximum error of 2%, how accurate is the volume measurement?
A sphere of radius R and diameter $x = 2R$ has volume

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{6}\pi x^3$$

so the volume using the estimated diameter $x = 2.5$cm is
$V = \frac{1}{6}\pi(2.5)^3 \approx 8.181$ cm^3. The error made in computing this volume using the diameter 2.5 when the actual diameter is $2.5 + \Delta x$ is

$$\Delta V = V(2.5 + \Delta x) - V(2.5) \approx V'(2.5)\Delta x$$

The measurement of the diameter can be off by as much as 2%, that is, by as much as $0.02(2.5) = 0.05$cm in either direction. Hence, the maximum error