Moreover $(1+\frac{1}{2}x)^{-1} \approx 1-\frac{1}{2}x$, (using $(1+u)^{-1} \approx 1-u$ with $u = x/2$). Hence;

$$\frac{e^{-2x}}{\sqrt{1+x}} \approx (1-2x)(1-\frac{1}{2}x) = 1-2x-\frac{1}{2}x+x^2$$

Now, we discard that last x^2 term,because we have already thrown out a number of other x^2 (and higher order) terms in making these approximations. Remember, were assuming that $|x| \ll 1$. This means that x^2 very small, x^3 even smaller, etc. We can ignore these higher-order terms, because they are very, very small. This yields

$$\frac{e^{-2x}}{\sqrt{1+x}} \approx 1-2x-\frac{1}{2}x = 1-\frac{5}{2}x$$

You can now quickly read from this approximation $f(0) = 1$ and $f'(0) = \frac{-5}{2}$

Example Find the limit $\lim\limits_{x\to 0} \frac{(1+2x)^{10}-1}{x}$ by approximating the quotient.

$$(1+2x)^{10} \approx 1+10(2x)(\text{ Use } (1+u)^r \approx 1+ru \text{ where } u = 2x \text{ and } r = 10)$$

So

$$\frac{(1+2x)^{10}-1}{x} = \frac{1+20x-1}{x} = 20$$

Example Approximate the value of $e^{0.5}$.
We will use the list we have created above and use $e^x \approx 1+x$. So
$e^{0.5} \approx 1+0.5 = 1.5$
Example Approximate the value of $\sqrt{4.1}$

Let $f(x) = \sqrt{x}$ so that were attempting to approximate $f(4.1) = \sqrt{4.1}$.Notice that I know the value of $f(x)$ and $f'(x)$ at $x = 4$, and so I can solve for the equation of the line tangent to the graph of $f(x)$ at $x = 4$. Specifically, since $f(4) = 2$ and $f'(4) = \frac{1}{4}$, the equation of the line tangent to $f(x)$ at $x = 4$ is

$$y-2 = \frac{1}{4}(x-4) \text{ or } y = \frac{1}{4}(x-4)+2$$

Now we use linearization: this tangent line is supposed to be a good approximation to $f(x)$ near $x = 4$, and so we can estimate $\sqrt{4.1}$ by plugging 4.1 into the equation of the tangent line. This gives

$$\sqrt{4.1} \approx \frac{1}{4}(4.1-4)+2 = 2\frac{1}{40}$$