then $\ln(1+u) \approx u$ with the new base point $u_0 = a - 1 = 0$.

Basic List of Linear Approximations

The following list of approximations is generated using $a = 0$ as the base point and assume that $|x| \ll 1$ (much smaller than 1).
1) $\sin x \approx x($ if $x \approx 0)$
2) $\cos x \approx 1($ if $x \approx 0)$
3) $e^x \approx 1 + x($ if $x \approx 0)$
4) $\ln(1+x) \approx x($ if $x \approx 0)$
5) $(1+x)^r \approx 1 + rx($ if $x \approx 0)$

Proofs Proof of 1) Take $f(x) = \sin(x)$ then $f'(x) = \cos(x)$, $f(0) = 0$ and $f'(0) = 1$

$$\sin(x) \approx f(0) + f'(0)(x - 0) = 0 + 1 \cdot (x) = x$$

The proofs of 2-3 are similar to the one above. We have proved 4th one in our first example.

Proof of 5) Let $f(x) = (1+x)^r$, then $f(0) = 1$ and $f'(x) = r(1+x)^{r-1} \Rightarrow f'(0) = r$. So ;

$$(1+x)^r \approx f(0) + f'(0)(x - 0) = 1 + rx$$

Example Find the linear approximation of $f(x) = \frac{e^{-2x}}{\sqrt{1+x}}$ near $a = 0$.

One way to go about is to do the approximation by finding $f(0)$ and $f'(0)$ and using the formula. I'll show you here another way by using the list we generated above:

$$e^{-2x} \approx 1 + (-2x)(e^u \approx 1 + u \text{ where } u = -2x)$$

by (3) above in the list.Then by (5)

$$\sqrt{1+x} = (1+x)^{1/2} \approx 1 + \frac{1}{2}x$$

Put these two approximation together:

$$\frac{e^{-2x}}{\sqrt{1+x}} \approx \frac{1-2x}{1+\frac{1}{2}x} = (1-2x)(1+\frac{x}{2})^{-1}$$