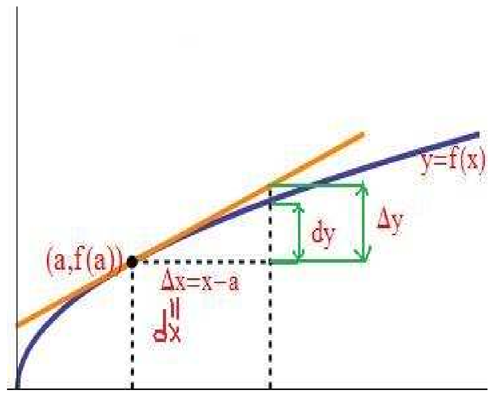
Definition The linear or tangent line approximation of $f(x)$ at "a" is

$$\boxed{f(x) \approx f(a) + f'(a)(x-a)}$$



Remarks: 1) This equation is mathematical way of writing what we have observed from the applet: The tangent line approximates f(x). Note that the equation of the tangent line(T) drawn to $y = f(x)$ at the point $(a, f(a))$ is

$$y_T(x) - f(a) = f'(a)(x-a) \Rightarrow y_T(x) = f(a) + f'(a)(x-a)$$

The tangent line approximation formula says, the function value at x i.e. $f(x)$ is very close to the value of x on the tangent line i.e. $y_T(x)$
2) The formula gives a good approximation near the tangent point "a". As you move away from "a"however, the approximation grows less accurate.
3) This approximation can be written alternatively in the following form;

$$\Delta y \approx dy = f'(a)\Delta x = f'(a)dx$$

where $\Delta y = f(x) - f(a)$ and $\Delta x = dx = x - a$. The term $dy = f'(a)\Delta x$ plays a special role in applied calculus and it is called the *differential of y.*

Example Find the linear approximation to $f(x) = \ln(x)$ at $a = 1$.

$f(1) = \ln(1) = 0$ also $f'(1) = \frac{1}{x}|_{x=1} = 1$ .So

$$\ln(x) \approx f(1) + f'(1)(x-1) = 0 + 1 \cdot (x-1) = x - 1$$

The approximation says for $x = 1$ and all nearby points you can think of $\ln(x)$ like the linear function $x - 1$. Here you may call the point a as the base point and you may change it as follows: Let $x = 1 + u \Rightarrow u = x - 1$