Section 3.1 Linear Approximations

Since the beginning of this course I have been talking about the "tangent problem". Now that we have solved the tangent problem, meaning we know how to write the equation of tangent line to any curve at any given point (*some restriction applies) I would like to get a use out of those tangent lines. First let's observe the relationship between the tangent line and the curve in the applet below

For the applet go to the web page : www.calculusapplets.com (and search for linear approximation).

Now the important observation: If we zoom in "enough" around the point $(a, f(a))$ to the curve, the curve and the tangent line becomes nearly inseperable from each other. This is not only true for the point $(a, f(a))$ but also for all nearby points. So the tangent to the graph of a function $f(x)$ at a point $(a, f(a))$ is a decent approximation to $f(x)$ at points near a.

This has a lot of great applications, but one of the most straightforward is that we can use the tangent line to $f(x)$ at "a" to approximate the values of $f(x)$ at points close to a. Since its frequently hard to evaluate functions at random points without a calculator, this will give us a technique to approximate certain quantities with only a little calculus know-how.

There are plenty of functions that we can evaluate easily. For instance, evaluating polynomials is not very difficult, since they only involve operations like addition, subtraction, and multiplication. For the same reason, computing the value of rational functions is also relatively easy. But is it easy to compute the value of functions like $\sqrt{x}$ or $\ln(x)$. It is not. We will use the tool we will introduce in this section to calculate them approximately.