Proof of MVT First let's write down the equation of the secant line. We have observed above that the slope of the secant line is $m_{sec} = \frac{f(b)-f(a)}{b-a}$. Since $(a, f(a))$ is a point on this line the secant line equation is

$$y_{sec} - f(a) = m_{sec}(x-a) \text{ or } y = m_{sec}(x-a) + f(a).$$

Now let $h(x) = f(x) - y_{sec}$ or by the above definition

$$h(x) = f(x) - m_{sec}(x-a) - f(a)$$

Next note

$$\begin{aligned} h(a) &= f(a) - m_{sec}(a-a) - f(a) \\ &= f(a) - 0 - f(a) = 0 \end{aligned}$$

and

$$\begin{aligned} h(b) &= f(b) - m_{sec}(b-a) - f(b) \\ &= f(b) - \left(\frac{f(b)-f(a)}{b-a}\right)(b-a) - f(a) \\ &= f(b) - (f(b) - f(a)) - f(a) = 0 \end{aligned}$$

So $h(a) = h(b) = 0$. Moreover $h(x)$ is differentiable because it is the difference of two differentiable functions.(One of the conditions on f is differentiability, y_{sec} is a polynomial and hence differentiable everywhere.). So $h(x)$ is continuous by a similar reasoning. Therefore, $h(x)$ satisfies all the following hypothesis of Rolle's Theorem:
i) h is continuous on $[a, b]$
ii)h is differentiable on (a, b)
iii) $h(a) = h(b) = 0$

Hence by Rolle's Theorem there is a "c" in the open interval (a, b) such that $h'(c) = 0$. What is $h'(x)$?

$$h'(x) = f'(x) - m_{sec} + 0$$

At "c"

$$h'(c) = f'(c) - m_{sec} = 0 \Rightarrow m_{sec} = f'(c) \Rightarrow \frac{f(b)-f(a)}{b-a} = f'(c)$$