Mean Value Theorem(MVT) If f is continuous on the closed interval [a, b] and differentiable on (a, b), then

$$\frac{f(b) - f(a)}{b - a} = f'(c) \text{ for some c in } (a, b)$$

Here what is going on geometrically:



Note that the quotient  $\frac{f(b)-f(a)}{b-a}$  is the slope of the secant line connecting the points (a, f(a)) and (b, f(b)). Mean Value Theorem (MVT) says that there is at least one c (maybe multiple as in the graph above) in the open interval (a, b) where the slope of the tangent line drawn to f is the same as the slope of the secant line.

**Cautionary** remark and example after Rolle's Theorem holds here as well. To be able to apply MVT you have to make sure first all conditions are satisfied. Also, MVT is another existential theorem, meaning it guarantees existence of a number c, it does not tell what it is. Furthermore just as in the graphical example above MVT does not guarantee uniqueness either, we might have couple of such "c"'s in the interval (a, b).

You can also interpret MVT <u>physically</u> as follows: If you are traveling from Champaign to Chicago and your average speed during the trip is 60mi/h, then MVT says at some point during your trip you had to be traveling at exactly 60mi/h.