Example Find all numbers "c" in the interval $(0, 2)$ that satisfy the conclusion to Rolle's Theorem for $f(x) = x^3 - 3x^2 + 2x + 5$

Following the "Caution" above let's first check whether the function f satisfies all the conditions for us to apply Rolle's or not.
a) First $f(x)$ is continuous everywhere because it is a polynomial hence it is continuous in particular over the closed interval $[0, 2]$
b) Also $f(x)$ is differentiable everywhere because of the same reason that it is a polynomial, hence over the open interval $(0, 2)$.
c) $f(0) = 0-0-0+5 = 5$ and $f(2) = 8-12+4+5 = 5$ implies $f(0) = f(2) = 5$

So we may apply Rolle's Theorem. And it guarantees that there is (or are) c(c's) in the open interval $(0, 2)$ such that $f'(c) = 0$. That is all. We have to find what they are by solving the equation
$f'(c) = 3c^2 - 6c + 2 = 0 \Rightarrow c = \frac{-(-6)\pm\sqrt{6^2-4(3)(2)}}{2(3)} = 1 \pm \sqrt{1 - \frac{2}{3}} = 1 \pm \sqrt{\frac{1}{3}}$

You have to make sure that both of these c values are in the interval $(0, 2)$ which is the case. So for $c_1 = 1 + \sqrt{\frac{1}{3}}$, $f'(c_1) = 0$ and also for $c_2 = 1 - \sqrt{\frac{1}{3}}$, $f'(c_2) = 0$
Our main purpose in introducing Rolle's Theorem is to prove the Mean Value Theorem even though Rolle's has other applications.