Example Compute $\frac{d}{dx} \arcsin x$

This doesn't look like an implicit differentiation problem, but we're going to make it one. We start by writing $y = \arcsin x$. Applying sine to both sides then gives $\sin(y) = \sin(\arcsin x) = x$. Now we can use implicit differentiation. The left hand side has derivative $\cos(y)\frac{dy}{dx}$ and the right hand side has derivative 1. This means we have

$$\cos(y)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$$

But what is $\cos(y)$ in terms of x? We know that $\cos^2(y) + \sin^2(y) = 1$ so that $\cos y = \sqrt{1 - \sin^2 y}$. But since $\sin(y) = x$, we have $\cos y = \sqrt{1 - x^2}$. Hence, we have

$$\frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1 - x^2}}$$

Example Compute $\frac{d}{dx} \arccos x$

Well use the same trick as last time. We start by writing $y = \arccos x$ and apply cosine to both sides to get $\cos(y) = \cos(\arccos x) = x$. The left hand side has derivative $-\sin(y)\frac{dy}{dx}$ and the right hand side has derivative 1. This means we have

$$-\sin(y)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = -\frac{1}{\sin y}$$

But what is sin(y) in terms of x? Again, we have $\cos^2(y) + \sin^2(y) = 1$ so that $\sin y = \sqrt{1 - \cos^2 y}$. But since $\cos(y) = x$, we have $\sin y = \sqrt{1 - x^2}$. Hence, we have

$$\frac{d}{dx} \arccos x = -\frac{1}{\sin(\arccos x)} = -\frac{1}{\sqrt{1 - x^2}}$$

Example Compute $\frac{d}{dx} \arctan x$

Try this yourself. You'll use the same ideas as before, but when simplifying this youll need the identity $1 + \tan^2 y = \sec^2 y$.