This examples embodies everything there is in implicit differentiation. To solve the tangent problem for a graph which isn't explicitly a function of x (i.e., in an expression which isn't "solved for y"), the technique is pretty simple:

• start with a complicated expression involving x's and y's; this makes y an implicit function of x;
• compute the derivative of both sides of this expression (don't forget those $\frac{dy}{dx}$'s which pop up!);
• solve for $\frac{dy}{dx}$;
• $\frac{dy}{dx}$ gives you the slope of the line tangent to the curve defined by your original complicated expression

Example Write the equation of the tangent line drawn to the curve $y^2 = x^3(2-x)$ at the point $(1,1)$.

We want to solve for $\frac{dy}{dx}$, so we need to compute the derivative of the left and right hand sides of the given equality. The right hand side has derivative

$$\frac{d}{dx}[x^3(2-x)] = \frac{d}{dx}[-x^4 + 2x^3] = 6x^2 - 4x^3$$

and the left hand side has derivative

$$\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$$

(remember: the $\frac{dy}{dx}$ is appearing because of our old friend the chain rule). Hence we have

$$2y\frac{dy}{dx} = 6x^2 - 4x^3$$

Solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = \frac{6x^2 - 4x^3}{2y} = \frac{3x^2 - 2x^3}{y}$$

This means the slope of the line tangent to the curve at $(1,1)$ is $\frac{3(1^2)-2(1^3)}{1} = 1$. Hence the line tangent to the curve at (1, 1) is

$$y - 1 = 1 \cdot (x-1) \Rightarrow y = x$$