Remark Logs are used in all sciences and even in finance. Think about the stock market. If I say the market fell 50 points today, youd need to know whether the market average before the drop was 300 points or 10, 000. In other words, you care about the percent change, or the ratio of the change to the starting value: $\frac{d}{dx}\ln(f(t)) = \frac{f'(t)}{f(t)}$

Example Differentiate $y = x^x$.

Note here not only the exponent is varying but also the base as well. For the derivative of such functions either use the "e" or logarithmic differentiation. I'll use the latter one and encourage you to try the other yourself (Hint: Write $f(x) = e^{x\ln x}$ and use Chain Rule):

$$
\begin{aligned}
f(x) &= x^x \\
\ln f &= \ln x^x = x\ln x \text{ (Take Derivative of both sides)} \Rightarrow \\
(\ln f)' &= 1\cdot\ln x + x\cdot\frac{1}{x} = \ln x + 1 \text{ by Product Rule}
\end{aligned}
$$

Now recall the logarithmic differentiation rule above and replace $(\ln f)'$ on the left hand side of the last equality with $(\ln f)' = \frac{f'(x)}{f(x)}$

$$\frac{f'(x)}{f(x)} = \ln x + 1 \Rightarrow f'(x) = f(x)(\ln x + 1) = x^x(\ln x + 1)$$

Example Find $\frac{d}{dx}\left(\ln(\frac{x+1}{\sqrt{x-2}})\right)$

One way to do this is to use the logarithmic differentiation with the Quotient Rule– try it yourself, it is little messy. I'll use the logarithm laws to simplify my function first then do the differentiation:

$$\ln(\frac{x+1}{\sqrt{x-2}}) = \ln(x+1)-\ln(\sqrt{x-2}) = \ln(x+1)-\ln(x-2)^{1/2} = \ln(x+1)-\frac{1}{2}\ln(x-2)$$

Now differentiate:

$$\frac{d}{dx}\left(\ln(\frac{x+1}{\sqrt{x-2}})\right) = \frac{d}{dx}\left(\ln(x+1) - \frac{1}{2}\ln(x-2)\right) = \frac{1}{x+1} - \frac{1}{2(x-1)}$$