Theorem Derivative of the **Natural Log Function** for $x > 0$ is

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$

Proof To prove this we will use the "Derivative of the Inverse Function" theorem wee have seen in Section 2.5 because we have just found the derivative of e^x and, e^x and $\ln x$ are inverses of each other. So;

$$\begin{aligned}\frac{d}{dx}\ln x &= \frac{1}{(\frac{d}{dx}e^x)|_{x=\ln x}} \\ &= \frac{1}{e^x|_{x=\ln x}} = \frac{1}{e^{\ln x}} = \frac{1}{x}\end{aligned}$$

Note that the argument above works on the domain of $\ln x$, i.e. for $x > 0$

Example Differentiate $y = \ln(x^3 + 1)$

Chain Rule is the way to go as y is composition of two functions again: $y = f(u) = \ln u$ and $u(x) = x^3 + 1$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx} \\ &= \frac{1}{u}\cdot 3x^2 \\ &= \frac{1}{x^3+1}\cdot 3x^2 = \frac{3x^2}{x^3+1}\end{aligned}$$

In the light of the example above I would like to introduce the

Logarithmic Differentiation.

The idea is to find $\frac{d}{dx}f(x)$ by finding $\frac{d}{dx}\ln(f(x))$ instead. Why? Because sometimes this approach is easier.Let $u = f(x)$ by Chain Rule:

$$\frac{d}{dx}\ln u = \frac{d(\ln u)}{du}\frac{du}{dx} = \frac{1}{u}(\frac{du}{dx})$$

Since $u = f$ and $\frac{du}{dx} = f'$, we can also write

$$\boxed{(\ln f)' = \frac{f'}{f} \text{ or } f' = f(\ln f)'}$$