Example At what point on the curve $y = e^x$ is the tangent line parallel to the line $y = 2x$?

$y = e^x \Rightarrow y' = e^x$

We want "x" such that $y'(x)$ = the slope of the tangent line = 2, because parallel lines have the same slopes. So solve the equation $e^x = 2$ for x:

$$\begin{aligned} e^x &= 2 \Rightarrow \\ \ln(e^x) &= \ln 2 \Rightarrow \\ x &= \ln 2 \end{aligned}$$

So at the point $(\ln 2, e^{\ln 2}) = (\ln 2, 2)$.

Example Find y' for $y = e^{2x}$.

This function is a composition of two function $e^{f(x)}$ where $f(x) = 2x$. So to finding derivative calls for Chain Rule:

$$\begin{aligned} y' &= \frac{d}{dx}(e^{2x}) \\ &= e^{2x} \cdot \frac{d}{dx}(2x) \\ &= e^{2x} \cdot 2 = 2e^{2x} \end{aligned}$$

<u>Remark</u> For $y = e^{f(x)}$ then $\boxed{y' = e^{f(x)} \cdot f'(x)}$

Theorem $\boxed{\frac{d}{dx} a^x = (\ln a) a^x}$

Proof Rewrite a as $e^{\ln a}$. Then

$$a^x = (e^{\ln a})^x = e^{x \ln a}$$

Now use the Remark above (which is a direct consequence of Chain Rule)

$$\frac{d}{dx} a^x = \frac{d}{dx} e^{x \ln a} \underbrace{=}_{f(x) = x \ln a} e^{x \ln a} (x \ln a)' = e^{x \ln a} (\ln a) = a^x (\ln a)$$