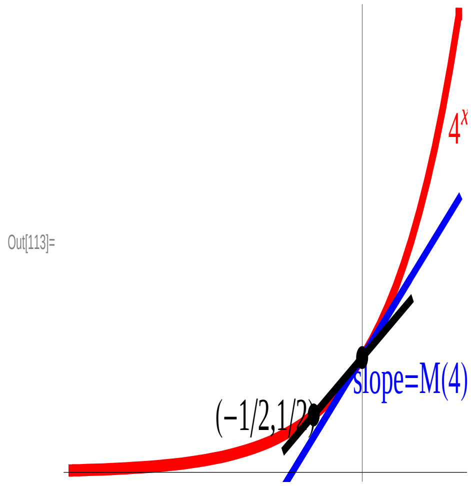
Next, look at the graph of 4^x below



The secant line from $(\frac{-1}{2}, \frac{1}{2})$ to $(0, 1)$ on the graph of $y = 4^x$ has slope 1. Therefore, the slope of $y = 4^x$ is less: at $x = 0$ is greater: $M(4) > 1$

Somewhere in between 2 and 4 there is a base whose slope at $x = 0$ is 1.

Thus we can define "e" to be the unique number such that $M(e) = 1$ or to put it another way $\lim\limits_{h\to 0} \frac{e^h-1}{h} = 1$ or $\frac{d}{dx}e^x = 1$ at $x = 0$

Theorem Derivative of the (Natural) **Exponential Function** is:

$$\boxed{\frac{d}{dx}e^x = e^x}$$

Proof After our definition above the proof is really easy now:

$$\begin{aligned}
\frac{d}{dx}e^x &= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\
&= e^x \left(\lim_{h\to 0} \frac{e^h - 1}{h} \right) \\
&= e^x \cdot 1 \text{ (by definition of "e")} \\
&= e^x
\end{aligned}$$