Section 2.7 Derivative of Exponential and Logarithmic Function

In this section we will be interested in finding the derivative of $\frac{d}{dx}a^x$ for any base "a". Let's first use the definition of the derivative and see where it leads us:

$$\frac{d}{dx}a^x = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = \lim_{h\to 0} a^x\frac{a^h-1}{h} = a^x\lim_{h\to 0}\frac{a^h-1}{h} \quad (\star)$$

Let's call $M(a) = \lim\limits_{h\to 0}\frac{a^h-1}{h}$. We don't know what $M(a)$ is yet but by the equation $\star$ we may write

$$\frac{d}{dx}a^x = M(a)a^x$$

Here are two ways to describe M(a):

- Using equation $(\star)$ we observe that $M(a) = \lim\limits_{h\to 0}\frac{a^{0+h}-a^0}{h} = \frac{d}{dx}a^x|_{x=0}$. So M(a) is the derivative of a^x at x=0.

- Geometrically M(a) is the slope of the graph $y = a^x$ at $x = 0$.

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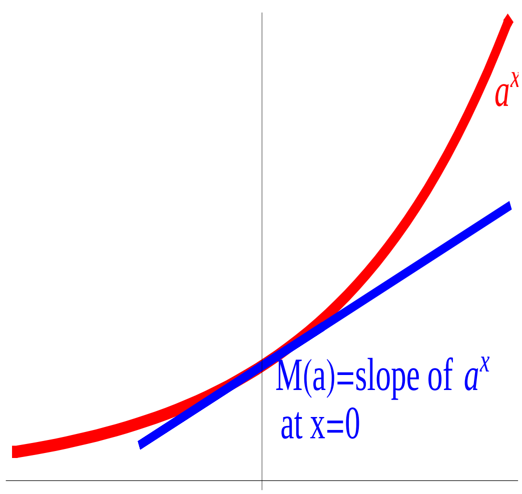

The trick to figuring out what M(a) is to define "e" as the number such that $M(e) = 1$. So there you go we just defined the number "e".