

You can similarly show the following is true:

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$$

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$

Example Find y' for $y = \tan(x)(x^3 - ex + 1)$

$$y' = \sec^2(x)(x^3 - ex + 1) + \tan(x)(3x^2 - e)$$

Example Find y' for $y = \sin(\cos(\tan x))$.

$$\begin{aligned} y' &= \cos(\cos(\tan x)) \cdot \frac{d}{dx}[\cos(\tan x)] \\ &= \cos(\cos(\tan x))[-\sin(\tan x)] \frac{d}{dx}[\tan x] \\ &= \cos(\cos(\tan x)) \cdot (-\sin(\tan x)) \cdot (\sec^2 x) \end{aligned}$$

Example Find the points on the curve $y = \frac{\cos(x)}{2+\sin(x)}$ at which the tangent is horizontal.

$$\begin{aligned} y' &= \frac{(2 + \sin x)(-\sin x) - (\cos x)(\cos x)}{(2 + \sin x)^2} \\ &= \frac{-2 \sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2} \\ &= \frac{-2 \sin x - 1}{(2 + \sin x)^2} \end{aligned}$$

Tangent line is horizontal when $y' = 0$

$$y' = \frac{-2 \sin x - 1}{(2 + \sin x)^2} = 0 \Rightarrow -2 \sin x - 1 = 0 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6} + 2\pi k \text{ or } x = \frac{11\pi}{6} + 2\pi k$$