Trigonometric Derivatives

Theorem $\boxed{\frac{d}{dx}[\sin(x)] = \cos(x)}$

Proof

$$
\begin{aligned}
(\sin(x))' &= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} \\
&= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h} \\
&= \lim_{h\to 0} \left(\frac{\sin(x)\cos(h) - \sin(x)}{h} + \frac{\sin(h)\cos(x)}{h} \right) \\
&= \sin(x) \lim_{h\to 0} \left(\frac{\cos(h) - 1}{h} \right) + \cos(x) \lim_{h\to 0} \left(\frac{\sin(h)}{h} \right) \\
&= \sin(x) \cdot 0 + \cos(x) \cdot 1 = \cos(x)
\end{aligned}
$$

Theorem $\boxed{\frac{d}{dx}[\cos(x)] = -\sin(x)}$

Proof Similar like the above one. This is a good exercise for you to try.

Once you have the derivative of $\sin(x)$ and $\cos(x)$, we have all the other trig derivatives:

Theorem $\boxed{\frac{d}{dx}[\tan(x)] = \sec^2(x)}$

Proof

$$
\begin{aligned}
\frac{d}{dx}[\tan(x)] &= \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] \quad \text{(Use Quotient Rule)} \\
&= \frac{\cos x \cos x - \sin x(-\sin x)}{[\cos^2 x]} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} = \sec^2 x
\end{aligned}
$$