Proof There are two cases. If $g(x) \neq g(a)$, then (by (2))

$$F(g(x)) = \frac{f(g(x)) - f(g(a))}{g(x) - g(a)}$$

so that

$$F(g(x)) \cdot \frac{g(x) - g(a)}{x - a} = \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \cdot \frac{g(x) - g(a)}{x - a} = \frac{f(g(x)) - f(g(a))}{x - a}$$

On the other hand, if $g(x) = g(a)$, then in (4), both (E) and (D) equal zero, while $F(g(x)) = f'(g(a))$; thus in this case (4) just says "$0 = 0$"

Now using the Lemma and the discussion above it we can have the "flawless" proof of Chain Rule as follows:

$$\begin{aligned}
(f \circ g)'(a) &= \lim_{x \to a} \frac{f(g(x)) - f(g(a))}{x - a} \\
&\underbrace{=}_{by\ (4)} \lim_{x \to a} F(g(x)) \cdot \frac{g(x) - g(a)}{x - a} \\
&= \lim_{x \to a} F(g(x)) \cdot \lim_{x \to a} \frac{g(x) - g(a)}{x - a} \\
&\underbrace{=}_{by\ (3)} f'(g(a)) \cdot g'(a)
\end{aligned}$$