Example Let $f(x) = x^2$ for $x \geq 0$, then $g(x) = f^{-1}(x) = \sqrt{x}$ for $x \geq 0$. We want to find $g'(x)$ and by theorem we need $f'(x) = 2x$. So

$$(f^{-1}(x))' = g'(x) = \frac{1}{f'(g(x))} = \frac{1}{2g(x)} = \frac{1}{2\sqrt{x}}$$

For this problem you can also compute the answer by finding the derivative of g(x) using Power Rule as follows:

$$g'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Example Let $x = \sqrt{y} + 5$. Find $\frac{dy}{dx}$

Since our problem is given x in terms of y I'll find $\frac{dx}{dy}$ first and use the theorem above to find $\frac{dy}{dx}$

$$\frac{dx}{dy} = \frac{1}{2}y^{-1/2} = \frac{1}{2\sqrt{y}}$$

Since $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ by the theorem, we have:

$$\frac{dy}{dx} = 2\sqrt{y} = 2(x-5)$$

Example Air is being pumped into a spherical balloon at the constant rate of $200\pi\ cm^3/s$. What is the rate of increase of the radius r when $r = 5$ cm. (Hint: The volume of a sphere $V = \frac{4\pi r^3}{3}$)

In this problem, both the volume and the radius are changing in time t, i.e $V(t) = \frac{4\pi(r(t))^3}{3}$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{4\pi(r(t))^3}{3}\right] = \frac{dV}{dr} \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

In the problem we are given $\frac{dV}{dr} = 200\pi\ cm^3/s$, and at this moment of time, $r = 5$ cm. Using this info and solving the above expression for $\frac{dr}{dt}$:

$$200\pi \text{ cm}^3/s = 4\pi(5 \text{ cm})^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{200\pi \text{ cm}^3/s}{100\pi \text{ cm}^2} = 2 \text{ cm}/s$$