The Flaw of the Proof above The problem with this proof is this: I can't be sure that I did not multiply and divide by zero. The definition of limit guarantees that (as $x \to a$) x will not equal a; but it cannot guarantee that $g(x) \neq g(a)$. This, after all, depends on the what function g does. Let me emphasize that this is the only flaw in the proof; if g happens to be a function for which $g(x) \neq g(a)$ whenever $x \neq a$, then the proof above is perfectly valid: by making g(x) sufficiently close to g(a) (which can be done by choosing x close to a), you can make $\frac{f(g(x))-f(g(a))}{x-a}$ as close as you like to $f'(g(a))$.
For those interested readers I am attaching the "How to correct the flaw" discussion at the end of these notes. I know the "correction proof" requires an acquired taste for proof and I do not expect everyone to look at it.

Example Find $F'(x)$ when $F(x) = \sqrt[3]{x^2-1}$

De-compose your functions $F(x) = f(g(x))$; where "outer" function
$f(g) = g^{1/3}$ and "inner" function $g(x) = x^2 - 1$. Find the derivatives of each of these functions as you'll need it for the Chain Rule.
$f'(g) = \frac{df}{dg} = \frac{1}{3}g^{-2/3}$
$g'(x) = \frac{dg}{dx} = 2x$
Use the Chain Rule now to find
$F'(x) = f'(g(x)) \cdot g'(x) = \frac{1}{3}(x^2-1)^{-2/3} \cdot 2x = \frac{2x}{3(x^2-1)^{2/3}}$

Example Let $y = (x^3-1)^{100}$ and find $\frac{dy}{dx}$

Let $y = u^{100}$ and $u(x) = x^3 - 1$. By Chain Rule (Leibnitz notation)
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
So again find each of the derivatives on the right hand side above and you are done.

$$\frac{dy}{du} = 100u^{99} \text{ and } \frac{du}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = 100(x^3-1)^{99} \cdot 3x^2 = 300x^2(x^3-1)$$

Example Let $f(x) = \frac{1}{\sqrt[5]{x^2+x+1}} = (x^2+x+1)^{-1/5}$. Find $f'(x)$?

$f(x) = u(v(x))$ where $u(v) = v^{-1/5}$ and $v(x) = x^2 + x + 1$.

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx} = -\frac{1}{5}(x^2+x+1)^{-6/5}(2x+1) = \frac{-(2x+1)}{5(x^2+x+1)^{6/5}}$$