

Let's move towards a more general formula.

Assume that $f(x)$ is differentiable. Let's calculate $\frac{d}{dx}(f(x))^3$

$$\begin{aligned}\frac{d}{dx}(f(x))^3 &= \frac{d}{dx}[f(x)f(x)f(x)] \\ &= f'(x)f(x)f(x) + f(x)f'(x)f(x) + f(x)f(x)f'(x) \\ &= 3 \cdot (f(x))^2 \cdot f'(x)\end{aligned}$$

Theorem(The Generalized Power Rule) If $f(x)$ is differentiable and n is a positive integer, then

$$\boxed{\frac{d}{dx}(f(x))^n = n(f(x))^{n-1}f'(x)}$$

For proof use the idea that I have used for $(f(x))^3$ above and prove it yourself.

Example $\frac{d}{dx}((x^3 - \pi)^7)$

Here the "outer" function is $f(u) = u^7$ and the "inner" function is $u(x) = x^3 - \pi$

$$\begin{aligned}\frac{d}{dx}[(x^3 - \pi)^7] &= (\text{derivative of the outer function at } u(x)) \cdot (\text{derivative of the inner function}) \\ &= 7(u)^6 \cdot (3x^2) \\ &= 7(x^3 - \pi)^6 \cdot (3x^2)\end{aligned}$$

Above investigation and the rule that followed only helps us to find the derivative of composition of two functions one of which is a power function such as $f(u) = u^7$ above. What about a general composition of two arbitrary functions?