Quotient Rule

Theorem (The Quotient Rule) Suppose f and g are differentiable functions. Then

$$\boxed{\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}}$$

Caution: Notice that in the formula above, it is VERY important that the function in the numerator is called f(x) and the function in the denominator is called $g(x)$; the minus sign in our expression will give us problems if we switch these names around! So be careful!

Proof Here is a nice and slick proof of the Quotient Rule that uses the Product Rule we have already proved instead of the long derivative definition type of proof.
We will define $H(x) = \frac{f(x)}{g(x)}$ so we are after $H'(x)$. First solve this for f(x); $f(x) = H(x)g(x)$ and then use the Product Rule :

$$\begin{aligned} f'(x) &= (g(x)H(x))' \\ &= g'(x)H(x) + g(x)H'(x) \end{aligned}$$

we want $H'(x)$ so solve for that term $\Rightarrow$

$$\begin{aligned} g(x)H'(x) &= f'(x) - g'(x)H(x) \\ H'(x) &= \frac{f'(x) - g'(x)H(x)}{g(x)} \\ &= \frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)} \\ &= \frac{\frac{f'(x)g(x) - g'(x)f(x)}{g(x)}}{g(x)} \\ &= \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \end{aligned}$$

Example Let $y = \frac{x^2+x-2}{x^3+6}$, find y'

$f(x) = x^2 + x - 2 \Rightarrow f'(x) = 2x + 1$
$g(x) = x^3 + 6 \Rightarrow g'(x) = 3x^2$
$y' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} = \frac{(2x+1)(x^3+6) - 3x^2(x^2+x-2)}{(x^3+6)^2}$