Example Differentiate $s(t) = \sqrt{t}(1-t)$.
Here $f(t) = \sqrt{t} = t^{1/2}$ and $g(t) = 1-t$. Since the Product Rule calls for the derivatives of these two functions let's calculate those first: $f'(t) = \frac{1}{2}t^{-1/2}$ and $g'(t) = -1$. So;

$$\begin{aligned} s'(t) &= f'(t)g(t) + f(t)g'(t) \\ &= \frac{1}{2}t^{-1/2}(1-t) + t^{1/2}(-1) \\ &= \frac{1}{2}t^{-1/2} - \frac{1}{2}t^{1/2} - t^{1/2} \\ &= \frac{1}{2\sqrt{t}} - \frac{3}{2}\sqrt{t} \end{aligned}$$

<u>Remark</u> If you don't have to use Product Rule, **DON'T**!

Go back to the example above and re-write $s(t) = t^{1/2} - t^{3/2}$ then use the power rule combined with the derivative rules we have seen in Section 2.3 to get

$$s'(t) = \frac{1}{2}t^{-1/2} - \frac{3}{2}t^{1/2} = \frac{1}{2\sqrt{t}} - \frac{3}{2}\sqrt{t}$$

Example If $f(x) = \sqrt{x}g(x)$ where $g(4) = 2$ and $g'(4) = 3$, find $f'(4)$.

$$\begin{aligned} f'(x) &= (\sqrt{x})'g(x) + \sqrt{x}g'(x) \\ &= \frac{1}{2\sqrt{x}}g(x) + \sqrt{x}g'(x) \end{aligned}$$

Now set $x = 4$

$$\begin{aligned} f'(4) &= \frac{1}{2\sqrt{4}}g(4) + \sqrt{4}g'(4) \\ &= \frac{1}{4} \cdot 2 + 2 \cdot 3 = \frac{13}{2} \end{aligned}$$

Example Evaluate $\frac{d}{dx}[f(x)g(x)h(x)]$

This is a little tricky, because the product rule only tells you how to compute the derivative of a product of two functions. To use it in this case, write $F(x) = f(x)g(x)$, then realize that you are evaluating $\frac{d}{dx}[F(x)h(x)]$. I will leave the rest to you to calculate the derivative of multiplication of three function with this hint.