

Once again the last equality is due the fact that we assumed  $f$  to be differentiable.

**Example**  $f(x) = 4x^5 + 3x^2\sqrt{x} - \frac{1}{x}$  find  $f'(x)$ .

First write the function as the sum of power functions after all those are the only ones so far we know how to differentiate.

$$f(x) = 4x^5 + 3x^{5/2} - x^{-1}$$

$$\begin{aligned} f'(x) &= (x^4)' + (3x^{5/2})' - (x^{-1})' \text{ by General Derivative Rules 1} \\ &= (x^4)' + 3(x^{5/2})' - (x^{-1})' \text{ by General Derivative Rules 2} \\ &= 4 \cdot 5x^4 + 3 \cdot \frac{5}{2}x^{3/2} - (-1)x^{-2} \\ &= 20x^4 + \frac{15}{2}x^{3/2} + \frac{1}{x^2} \end{aligned}$$

**Example**  $f(x) = \frac{x^2+4x+3}{\sqrt{x}}$  Find  $f'(x)$

Once again first write the function as the sum of power functions first;

$$f(x) = \frac{x^2}{\sqrt{x}} + \frac{4x}{\sqrt{x}} + \frac{3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2}$$

$$\begin{aligned} f'(x) &= (x^{3/2})' + (4x^{1/2})' + 3(x^{-1/2})' \text{ by General Derivative Rules 1} \\ &= (x^{3/2})' + 4(x^{1/2})' + 3(x^{-1/2})' \text{ by General Derivative Rules 2} \\ &= \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2} \\ &= \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x^{3/2}} \end{aligned}$$

### Higher Derivatives

If  $f$  is a differentiable function, then its derivative  $f'$  is also a function, so  $f'$  may have a derivative of its own.

**Example** Let  $f(x) = x^3 - 6x^2 - 5x + 3$ . By derivative rules and the power rule we know:

$$\frac{df}{dx} = 3x^2 - 12x - 5$$

$$\frac{d^2f}{dx^2} = 6x - 12$$

$$\frac{d^3f}{dx^3} = 6,$$

$$\frac{d^4f}{dx^4} = 0$$

Note that  $\frac{d^n f}{dx^n} = 0$  for all  $n \geq 4$