

**Example** For  $f(x) = \sqrt[3]{x^4} = x^{4/3}$ , find  $f'(x)$ .  
 $f'(x) = \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{1/3}$

**Example** For  $g(x) = x^\pi \Rightarrow g'(x) = \pi x^{\pi-1}$

**Theorem (General Derivative Rules)** If  $f(x)$  and  $g(x)$  are differentiable at  $x$ , and  $c$  is any constant then,

$$1) \frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

$$2) \frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)]$$

**Proof** 1) Let  $y = f(x) + g(x)$

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[f(x+h) - f(x)]}{h} + \lim_{h \rightarrow 0} \frac{[g(x+h) - g(x)]}{h} \\ &= \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \end{aligned}$$

The last equality above is due the fact that both individual limits exists because we assumed  $f$  and  $g$  to be differentiable.

You can prove the  $y = f(x) - g(x)$  case the same way I did above.

2) Let  $y = cf(x)$ .

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{cf(x+h) - cf(x)}{h} \\ &= \lim_{h \rightarrow 0} c \left[ \frac{f(x+h) - f(x)}{h} \right] \\ &= c \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right] \text{ by Limit Law 2} \\ &= c \frac{d}{dx}[f(x)] \end{aligned}$$