$$
\begin{aligned}
\frac{d}{dx}(x^n) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{(x+h)^n-x^n}{h}\\
&= \lim_{h\to 0}\frac{x^n+nx^{n-1}h+\frac{n(n-1)}{2}x^{n-2}h^2+\ldots+\binom{n}{k}x^{n-k}h^k+\ldots nxh^{n-1}+h^n-x^n}{h}\\
&= \lim_{h\to 0}\frac{nx^{n-1}h+\frac{n(n-1)}{2}x^{n-2}h^2+\ldots+\binom{n}{k}x^{n-k}h^k+\ldots nxh^{n-1}+h^n}{h}\\
&= \lim_{h\to 0} nx^{n-1}+\underbrace{\frac{n(n-1)}{2}x^{n-2}h+\ldots+\binom{n}{k}x^{n-k}h^{k-1}+\ldots nxh^{n-1}+h^{n-1}}_{\text{there is an h in every term here}}\\
&= nx^{n-1}+0+0+\ldots.+0=nx^{n-1}
\end{aligned}
$$

**Example** Let $f(x)=x^{2008}$ then by *Power Rule* $f'(x)=2008x^{2008-1}=2008x^{2007}$

The above theorem is true for integers. What if the power is a fraction? Let's check on an example;

**Example** Let $f(x)=\sqrt{x}$ and find $f'(x)$ ?

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\\
&= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\\
&= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}=\frac{1}{2\sqrt{x}}=\frac{1}{2}x^{-1/2}
\end{aligned}
$$

So $f(x)=x^{1/2}$ gives $f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2}x^{1/2-1}$ which is like the Power Rule.

In fact the Power Rule holds for any real number as we will state in the next theorem. For the proof of this theorem you'll need to wait until "logarithmic" differentiation though.

**Theorem (The General Power Rule)** For any real number r,

$$\boxed{\frac{d}{dx}(x^r)=rx^{r-1}}$$