Section 2.3 The Power Rule

Let's first find some common derivatives

Theorem For any constant c, $\boxed{\frac{d}{dx}(c) = 0}$

Proof Let $f(x) = c$

$$\begin{aligned} \frac{d}{dx}(c) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0 \end{aligned}$$

Theorem Let $f(x) = x$, then $\boxed{\frac{d}{dx}(x) = 1}$

Proof

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{(x+h) - x}{h} = \lim_{h \to 0} 1 = 1 \end{aligned}$$

Theorem (The Power Rule) For any integer $n > 0$, if $f(x) = x^n$, then

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

Proof The proof of this theorem requires us to recall the <u>Binomial Theorem</u> from PreCalc:

$$(x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + ... + \binom{n}{k}x^{n-k}h^k + ...nxh^{n-1} + h^n$$

where $\binom{n}{k} = \frac{n(n-1)...(n-k+1)}{1 \cdot 2 \cdot 3 ... k}$